# An Examination of the Taylor Standard Series of Hull Forms[1]


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## 1. Introduction

As a result of a previous re-analysis, the Taylor Standard Series is described in [1]. For the present we are interested in the hull shape and its value in simulations and not particularly in the resistance coefficients, though these could be important at some point. The original parent form was based on the British armored cruiser, the "Leviathan" of the Drake Class built in 1900. Therefore the series is appropriate to twin screws and a cruiser type of stern. The original has undergone some significant changes and now the model has a 3% bulb at the bow and the forefoot has dropped to the baseline. The maximum section has been moved to mid-length from station 19.2 to station 20.

In [1], the parameters that are varied across the models are the ratio of the beam to the draft, the longitudinal prismatic coefficient, $C_p$, and the displacement to length ratio. The prismatic coefficient is given by:

$$C_p = \frac{V}{A_x L} \quad , \tag{1}$$

where $V$ is the immersed volume, $A_x$ is the cross-sectional area at mid-length and $L$ is the waterline length. The displacement to length coefficient, $C_V$, is given by:

$$C_V = \frac{V}{L^3} \, . \tag{2}$$

In the re-analysis, the beam-to-draft ratios of the models were 2.25, 3.00, and 3.75. Prismatic coefficients ranged from 0.48 to 0.8 and, by extrapolation to 0.86.

To derive the various offspring models from the parent the mid-ships coefficient was held constant. The mid-ships coefficient, $C_x$, is given by:

$$C_x = \frac{A_x}{BH} \, , \tag{3}$$

where $B$ is the beam and $H$ is the draft. Variations in $C_p$ are introduced using the non-dimensional formula that can be derived directly from the definitions in (1) to (3):

$$\left(\frac{L}{B}\right)^2 = \frac{C_x C_p}{C_V \dfrac{B}{H}} \, . \tag{4}$$

The offsets are plotted in Figs 1 and 2 to ensure that the table of the parent offsets does not contain errors. (This was accomplished by entering the offsets into Excel and using

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the smoothed plotting option.) In this case 3 improvements were made to smooth out minor irregularities in the first and third station data (bow section). Other minor improvements could have been implemented but were not.

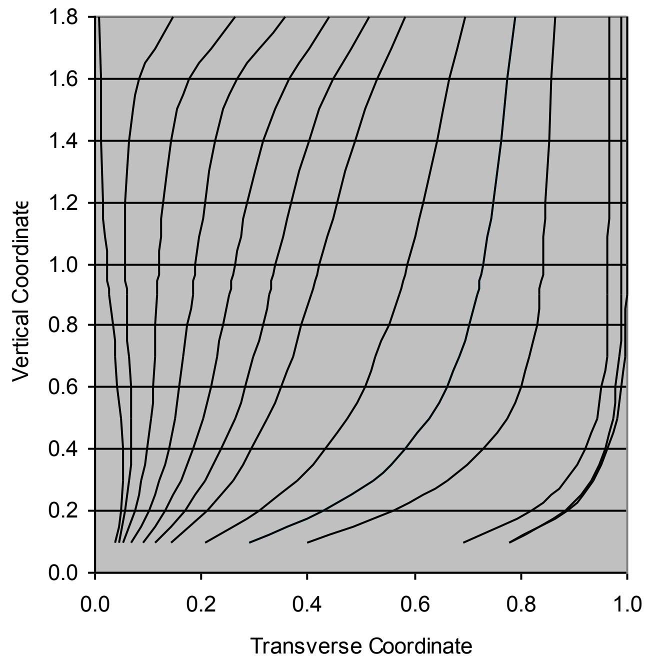


Figure 1. Bow section offsets from adjusted table.

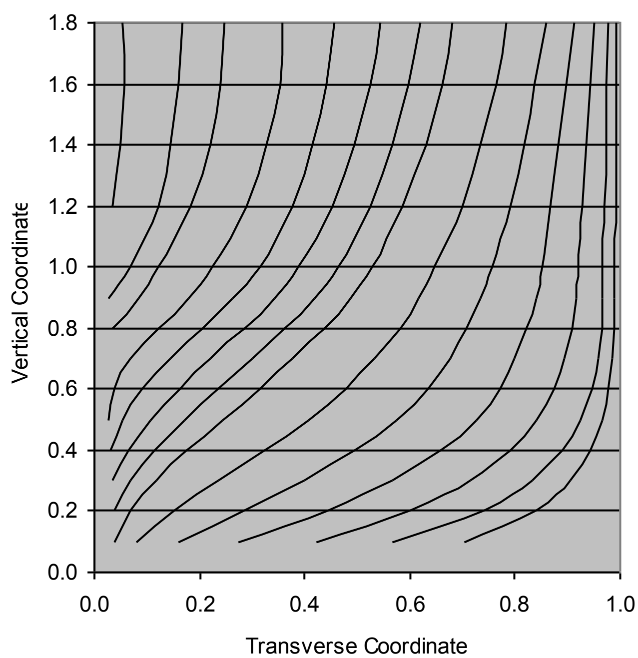


Figure 2. Stern section offsets from table.

A table of offsets represents the hull shape at a finite number of longitudinal positions and depths. Therefore it does not uniquely define the hull shape. This can be overcome by specifying an interpolation algorithm. In the following we assume that cubic splines are appropriate.

## 2. Offspring Models

The models in the series are derived from the parent using a non-linear transformation, which stretches and compresses the parent hull form in the longitudinal direction. The shapes of the transverse sections are preserved during this transformation so that it can be summarized by a set of curves that represent the cross section of a model in the series as a function of the longitudinal distance, *x*, with the overall prismatic coefficient as a parameter. (The bow and stern prismatic coefficients are similar to one another in all offspring.) Fig 3 shows the curves from Gertler [1]. The maximum half beam, *y*, has been normalized to unity. As noted previously, the mid-ships point is now half way between bow and stern perpendiculars; “FP” and “AP” denote the forward and aft perpendiculars respectively.

According to [1], the series model sectional areas for the bow and the stern can be described by a fifth order polynomial:

$$y = Q + C_p P + tT + nN + fF \,. \qquad (5)$$

Here $Q$, $P$, $T$, $N$ and $F$ are themselves fifth order polynomials in $x$, the normalized longitudinal distance measured from the bow or the stern to mid-ships, where $x = 1$. The parameter $t$ is the slope at $x = 0$, $n$ is the second derivative at $x = 1$ and $f$ is related to a bulbous bow or transom stern. The coefficients are given in [1]:

$$\begin{aligned} Q &= -30x^2 + 100x^3 - 105x^4 + 36x^5 \\ P &= 60x^2 - 180x^3 + 180x^4 - 60x^5 \\ T &= x - 6x^2 + 12x^3 - 10x^4 + 3x^5 \\ N &= -0.5x^2 + 2x^3 - 2.5x^4 + x^5 \\ F &= 1 - 30x^2 + 80x^3 - 75x^4 + 25x^5 \end{aligned} \quad . \qquad (6)$$

However, the expression for $F$ should be:

$$F = 1 - 30x^2 + 80x^3 - 75x^4 + 24x^5 \,. \qquad (7)$$

This is because we now have $F(0) = 1$, $F(1) = F'(0) = F'(1) = F''(1) = 0$ (where primes indicate derivatives with respect to $x$). These relationships are derived partly from the requirement that the presence of a bulbous bow should not affect the shape of the hull near mid-ships. Also the tangent at the bow is unaffected by the bulb. Because there are now 5 constraints to apply to $F$, there is only one free parameter, other than $f$, to represent the effect of a bulbous bow (or transom stern). Otherwise, using the version of $F$ from [1], we find that the value of $y$ at mid-ships is not 1 but is $1 + f$, which is very undesirable.

The corrected formulae are applied separately to the bow and stern sections. The numerical coefficients in the remaining formulae are also subject to certain constraints. In the absence of a bulbous bow or transom stern, the first is that the cross-section is zero at the fore or aft perpendicular. This is obviously satisfied when $x = 0$. The second is that $y = 1$ at $x = 1$. Therefore, as expected we have $Q = 1$ and $P = T = N = 0$. Furthermore the slope at $x = 1$ is zero. This can easily be verified by differentiation. It can also be verified that the slope of $y$ at the origin is just $t$ and that the second derivative, $d^2y/dx^2$, at $x = 1$ is just $n$.

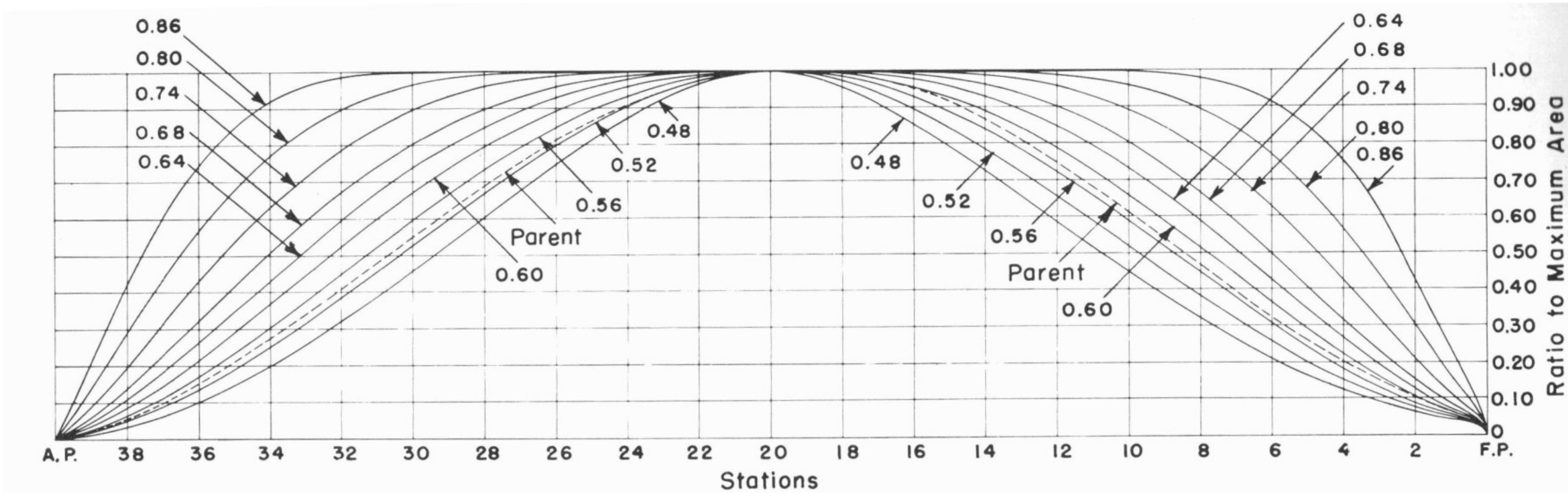


Figure 3. Sectional area curves from [1].

An arbitrary fifth order polynomial in $x$ has 6 coefficients. There are 3 fundamental constraints described previously. In addition the slope at $x = 0$ and the second derivative at $x = 1$ are specified. In principle, the remaining arbitrariness is removed by specifying $C_p$. Values for the coefficients $P$, $Q$, $P$, $T$, $N$ and $F$ have been calculated and the table 2 in [1] has been verified. The sectional area curves have also been calculated.

Plots in [1] provide values for $t$ and $n$ as a function of prismatic coefficient. To avoid reference to graphs and maintain an automatic digital approach, the plots of $t$ and $n$ are represented by fitting polynomials to the appropriate curves.

An example is shown in Fig 4, which shows some calculated sectional area curves for a plain stern. These are similar but are certainly not identical to those from [1] depicted in Fig 3. The plots in Fig 5 apply to a similar set of parameters, but with a 3 percent bulb at the bow ($f = 0.03$). It is worth noting that sometimes the maximum value of $y$ can be very slightly greater than one.

A comparison of the curves in Figs 3, 4 and 5 suggests that the approximation may start to fail quite badly for prismatic coefficients smaller than 0.52 (see later discussion). Alternatively, this entire approach may be regarded in part as an updated definition of the Taylor Standard hull forms.

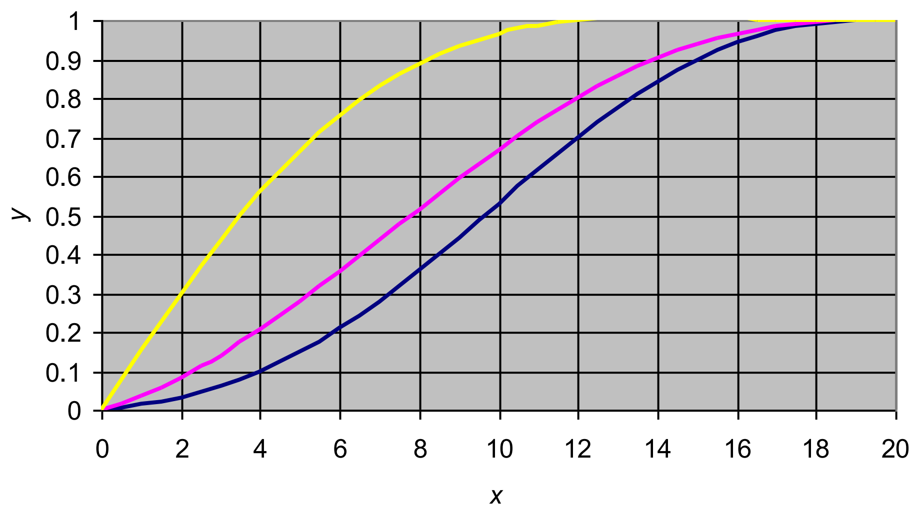


Figure 4. Calculated plain stern sectional area curves for $C_p$ values of 0.52 (-), 0.60 (-) and 0.80 (-).

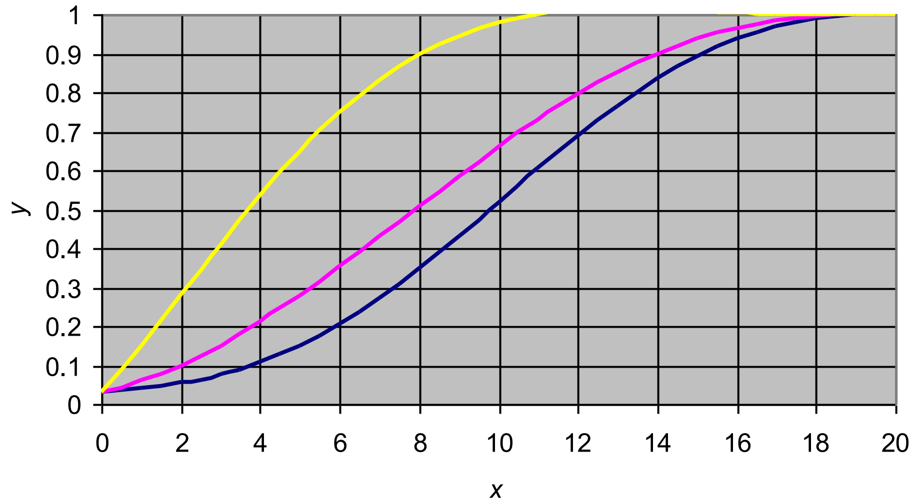


Figure 5. Calculated bulbous bow sectional area curves for $C_p$ values of 0.52 (-), 0.60 (-) and 0.80 (-).

## 3. An Example

In practice we would like to specify the waterline length and beam, the draft and the block coefficient. The prismatic coefficient and the volumetric coefficient are not so interesting. However, we do need the prismatic coefficient to derive the coefficients *t* and *n*. The block coefficient, $C_B$, is defined by:

$$C_B = V/(LBH)\,. \tag{8}$$

Combining this with (1) and (3) yields:

$$C_p = C_B / C_x\,. \tag{9}$$

It is noted in [1] that $C_x = 0.925$ for the Taylor series parent and its offspring. Therefore the prismatic coefficient is about 7.5% greater than the block coefficient. As an example, we consider the Olmeda used in the Loch Linnhe trials that took place in Scotland from 1989 to 1994. This ship is no longer in service. The approximate dimensions and displacement are shown in Table 1.

Table 1. Olmeda Characteristics.

| | |
|---|---|
| Type | Fleet Oiler |
| Length (m) | 180 |
| Beam (m) | 26 |
| Draft (m) | 9.2 |
| Displacement (tonnes) | 29,000 |

In fresh water the submerged volume is numerically equal to the displacement in metric tons. Therefore the block coefficient is about 0.67. If it is assumed that the Olmeda hull was similar to a Taylor series hull, the prismatic coefficient is about 0.73. We can determine the values of $t$ and $n$ from the graphs in [1] or an equivalent numerical method. For the bow, these are 1.22 and -0.03 respectively; for the stern they are 1.60 and -0.03. This allows the appropriate bow and stern sectional area curves to be calculated and these are then used to determine a new set of hull offsets derived from the parent.

## 4. Algorithm Details

In the parent hull, the prismatic coefficient of the bow section is 0.574 and that of the stern is 0.532. The average of these is 0.553. As noted already, the block coefficient can be derived from the prismatic coefficient by multiplying it by the mid-ships coefficient, which is 0.925. Therefore the block coefficient of the parent should be 0.512. As a check and using the table of offsets in [1], the block coefficient is calculated by numerically integrating over the submerged hull volume using the trapezoidal algorithm: this results in a value of 0.502, and is about 2% less than the expected value. The difference is due in part to the algorithm, which in this case tends to underestimate the volume.

The table of parent hull offsets is stored in an array (copied directly into the program from the Excel file to avoid typographical errors) along with vectors that represent the stations and their heights above the baseline (up to the load water line). The new value of block coefficient for the offspring is an input and from this the new prismatic coefficient is calculated. The values of $t$ and $n$ for both bow and stern sections are then found from the polynomial fits to the graphs in [1]. Then the hull sectional area curve is calculated as described above.

In general the transverse section of each station in the parent model is shifted along the longitudinal axis, $x$, according to the sectional area curves corresponding to both the parent and offspring. For each transverse section in the parent, its new position in the offspring is found by equating the ordinate, $y$, of the parent curve at a known parent station with the ordinate of the offspring curve at an unknown station. This is accomplished with a bisection search method over longitudinal distance $x$, in which the difference between the two $y$-values is brought very close to zero.

If needed, a set of new offspring offsets can be calculated at a pre-determined set of stations by a process of interpolation (using cubic splines). However, in this application a distribution of source and sink strengths is calculated. This source distribution represents the effect of the hull on the water flow around the hull. Using the “thin ship theory”, the sources are placed on the hull’s vertical centreplane running from bow to stern. It can be shown [2] that this distribution is proportional to the slope $dy/dx$. Fig 6 shows the slope distribution for a normalized hull for which the immersed volume occupies a unit cube; the block coefficient is 0.5. The slopes are shown for 25 longitudinal stations and 5 equally spaced depths at and below the water line (excluding the keel for which the slopes are zero). The curve appropriate to a depth can be identified by noting that, at the 5$^{th}$ station, the curves are in order of depth with the highest at the waterline.

Near the stern (stations 21 to 24), some slopes exhibit irregularities that appear to be associated with the necessity for creating space for the twin propellers. There are other minor irregularities that suggest that the hull may not be as streamlined as it seems as well as questions about the accuracy with which the parent hull was measured (or tabulated).

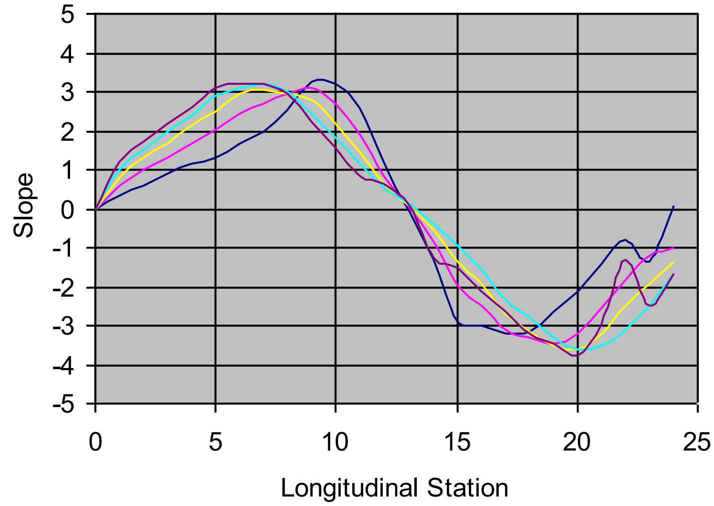


Figure 6. Taylor hull source distribution for 25 stations: $C_B = 0.5$.

The source distribution is an important consideration in wave making and consequently in wave making resistance. The wave amplitudes are approximately proportional to the magnitude of the sources. The distribution affects how individual wave components interfere. The distribution in Fig 6, which is spread out longitudinally, suggests that waves will often cancel and that wave making will be small over a wide range of speeds.

As noted earlier, the approximations adopted in [1] are apt to lead to significant differences between the block coefficient entered as input and the block coefficient of the achieved offspring. Table 2 shows a comparison between the two coefficients for hulls with a 3% bulb and rounded cruiser stern.

Table 2.
Comparison of Block Coefficients

| **Desired $C_B$** | **Actual $C_B$** |
|---|---|
| 0.50 | 0.47 |
| 0.55 | 0.52 |
| 0.56 | 0.53 |
| 0.60 | 0.57 |
| 0.65 | 0.62 |
| 0.70 | 0.67 |
| 0.73 | 0.70 |

## *5. Discussion*

The approximations adopted in [1] appear to lead to a block coefficient that is significantly less than that desired. However, it is likely that this is mainly due to the use of the trapezoidal method of integration that will tend to underestimate the immersed volume. The additional problems with small block coefficients that might have been expected do not occur. This implies that the approach in [1] is best regarded as an updated definition of the series. Though it would be possible to improve the method of integration involving the fitting of curves to the data points, it is questionable whether more effort on this topic is worthwhile; in any case there are several other more modern methodical series that are available.

When the block coefficient is high and for some ship dimensions (small length and large beam) the source distributions suggest that there could be some problems with boundary layer separation near the stern. Such hulls could be impractical owing to excessive resistance that this implies.

In spite of minor problems, the Taylor standard series is likely to be a useful component of simulations.

## References

[1] M. Gertler, *A reanalysis of the original test data for the Taylor Standard Series*, David W. Taylor Model Basin Report 806, Washington D.C., March 1954.

[2] A.E. Molland, S.R. Turnock and D.A. Hudson, *Ship Resistance and Propulsion*, Cambridge University Press, 2011.