

Table 1: Properties of the Continuous-Time Fourier Series

$$x(t) = \sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t} = \sum_{k=-\infty}^{+\infty} a_k e^{jk(2\pi/T)t}$$

$$a_k = \frac{1}{T} \int_T x(t) e^{-jk\omega_0 t} dt = \frac{1}{T} \int_T x(t) e^{-jk(2\pi/T)t} dt$$

Property	Periodic Signal	Fourier Series Coefficients
	$\left. \begin{array}{l} x(t) \\ y(t) \end{array} \right\} \begin{array}{l} \text{Periodic with period } T \text{ and} \\ \text{fundamental frequency } \omega_0 = 2\pi/T \end{array}$	$\begin{array}{l} a_k \\ b_k \end{array}$
Linearity	$Ax(t) + By(t)$	$Aa_k + Bb_k$
Time-Shifting	$x(t - t_0)$	$a_k e^{-jk\omega_0 t_0} = a_k e^{-jk(2\pi/T)t_0}$
Frequency-Shifting	$e^{jM\omega_0 t} = e^{jM(2\pi/T)t} x(t)$	$a_{k-M}$
Conjugation	$x^*(t)$	$a_{-k}^*$
Time Reversal	$x(-t)$	$a_{-k}$
Time Scaling	$x(\alpha t), \alpha > 0$ (periodic with period $T/\alpha$ )	$a_k$
Periodic Convolution	$\int_T x(\tau)y(t - \tau)d\tau$	$Ta_k b_k$
Multiplication	$x(t)y(t)$	$\sum_{l=-\infty}^{+\infty} a_l b_{k-l}$
Differentiation	$\frac{dx(t)}{dt}$	$jk\omega_0 a_k = jk \frac{2\pi}{T} a_k$
Integration	$\int_{-\infty}^t x(t)dt$ (finite-valued and periodic only if $a_0 = 0$ )	$\left(\frac{1}{jk\omega_0}\right) a_k = \left(\frac{1}{jk(2\pi/T)}\right) a_k$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} a_k = a_{-k}^* \\ \Re\{a_k\} = \Re\{a_{-k}\} \\ \Im\{a_k\} = -\Im\{a_{-k}\} \\  a_k  =  a_{-k}  \\ \angle a_k = -\angle a_{-k} \end{cases}$
Real and Even Signals	$x(t)$ real and even	$a_k$ real and even
Real and Odd Signals	$x(t)$ real and odd	$a_k$ purely imaginary and odd
Even-Odd Decomposition of Real Signals	$\begin{cases} x_e(t) = \mathcal{E}v\{x(t)\} & [x(t) \text{ real}] \\ x_o(t) = \mathcal{O}d\{x(t)\} & [x(t) \text{ real}] \end{cases}$	$\begin{array}{l} \Re\{a_k\} \\ j\Im\{a_k\} \end{array}$

Parseval's Relation for Periodic Signals

$$\frac{1}{T} \int_T |x(t)|^2 dt = \sum_{k=-\infty}^{+\infty} |a_k|^2$$

Table 2: Properties of the Discrete-Time Fourier Series

$$x[n] = \sum_{k=\langle N \rangle} a_k e^{jk\omega_0 n} = \sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n}$$

$$a_k = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk\omega_0 n} = \frac{1}{N} \sum_{n=\langle N \rangle} x[n] e^{-jk(2\pi/N)n}$$

Property	Periodic signal	Fourier series coefficients
	$x[n]$ } Periodic with period $N$ and fun- $y[n]$ } damental frequency $\omega_0 = 2\pi/N$	$a_k$ } Periodic with $b_k$ } period $N$
Linearity	$Ax[n] + By[n]$	$Aa_k + Bb_k$
Time shift	$x[n - n_0]$	$a_k e^{-jk(2\pi/N)n_0}$
Frequency Shift	$e^{jM(2\pi/N)n} x[n]$	$a_{k-M}$
Conjugation	$x^*[n]$	$a_{-k}^*$
Time Reversal	$x[-n]$	$a_{-k}$
Time Scaling	$x_{(m)}[n] = \begin{cases} x[n/m] & \text{if } n \text{ is a multiple of } m \\ 0 & \text{if } n \text{ is not a multiple of } m \end{cases}$ (periodic with period $mN$ )	$\frac{1}{m} a_k$ (viewed as periodic with period $mN$ )
Periodic Convolution	$\sum_{r=\langle N \rangle} x[r]y[n - r]$	$Na_k b_k$
Multiplication	$x[n]y[n]$	$\sum_{l=\langle N \rangle} a_l b_{k-l}$
First Difference	$x[n] - x[n - 1]$	$(1 - e^{-jk(2\pi/N)}) a_k$
Running Sum	$\sum_{k=-\infty}^n x[k]$ (finite-valued and periodic only if $a_0 = 0$ )	$\left( \frac{1}{(1 - e^{-jk(2\pi/N)})} \right) a_k$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} a_k = a_{-k}^* \\ \Re\{a_k\} = \Re\{a_{-k}\} \\ \Im\{a_k\} = -\Im\{a_{-k}\} \\  a_k  =  a_{-k}  \\ \angle a_k = -\angle a_{-k} \end{cases}$
Real and Even Signals	$x[n]$ real and even	$a_k$ real and even
Real and Odd Signals	$x[n]$ real and odd	$a_k$ purely imaginary and odd
Even-Odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}v\{x[n]\}$ [ $x[n]$ real] $x_o[n] = \mathcal{O}d\{x[n]\}$ [ $x[n]$ real]	$\Re\{a_k\}$ $j\Im\{a_k\}$

Parseval's Relation for Periodic Signals

$$\frac{1}{N} \sum_{n=\langle N \rangle} |x[n]|^2 = \sum_{k=\langle N \rangle} |a_k|^2$$

Table 3: **Properties of the Continuous-Time Fourier Transform**

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\omega) e^{j\omega t} d\omega$$

$$X(j\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t} dt$$

Property	Aperiodic Signal	Fourier transform
	$x(t)$	$X(j\omega)$
	$y(t)$	$Y(j\omega)$
Linearity	$ax(t) + by(t)$	$aX(j\omega) + bY(j\omega)$
Time-shifting	$x(t - t_0)$	$e^{-j\omega t_0} X(j\omega)$
Frequency-shifting	$e^{j\omega_0 t} x(t)$	$X(j(\omega - \omega_0))$
Conjugation	$x^*(t)$	$X^*(-j\omega)$
Time-Reversal	$x(-t)$	$X(-j\omega)$
Time- and Frequency-Scaling	$x(at)$	$\frac{1}{ a } X\left(\frac{j\omega}{a}\right)$
Convolution	$x(t) * y(t)$	$X(j\omega)Y(j\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} X(j\omega) * Y(j\omega)$
Differentiation in Time	$\frac{d}{dt}x(t)$	$j\omega X(j\omega)$
Integration	$\int_{-\infty}^t x(t) dt$	$\frac{1}{j\omega} X(j\omega) + \pi X(0)\delta(\omega)$
Differentiation in Frequency	$tx(t)$	$j \frac{d}{d\omega} X(j\omega)$
Conjugate Symmetry for Real Signals	$x(t)$ real	$\begin{cases} X(j\omega) = X^*(-j\omega) \\ \Re\{X(j\omega)\} = \Re\{X(-j\omega)\} \\ \Im\{X(j\omega)\} = -\Im\{X(-j\omega)\} \\  X(j\omega)  =  X(-j\omega)  \\ \angle X(j\omega) = -\angle X(-j\omega) \end{cases}$
Symmetry for Real and Even Signals	$x(t)$ real and even	$X(j\omega)$ real and even
Symmetry for Real and Odd Signals	$x(t)$ real and odd	$X(j\omega)$ purely imaginary and odd
Even-Odd Decomposition for Real Signals	$x_e(t) = \mathcal{E}v\{x(t)\}$ $[x(t)$ real] $x_o(t) = \mathcal{O}d\{x(t)\}$ $[x(t)$ real]	$\Re\{X(j\omega)\}$ $j\Im\{X(j\omega)\}$

Parseval's Relation for Aperiodic Signals

$$\int_{-\infty}^{+\infty} |x(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} |X(j\omega)|^2 d\omega$$

Table 4: Basic Continuous-Time Fourier Transform Pairs

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=-\infty}^{+\infty} a_k e^{jk\omega_0 t}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta(\omega - k\omega_0)$	$a_k$
$e^{j\omega_0 t}$	$2\pi \delta(\omega - \omega_0)$	$a_1 = 1$ $a_k = 0$ , otherwise
$\cos \omega_0 t$	$\pi[\delta(\omega - \omega_0) + \delta(\omega + \omega_0)]$	$a_1 = a_{-1} = \frac{1}{2}$ $a_k = 0$ , otherwise
$\sin \omega_0 t$	$\frac{\pi}{j}[\delta(\omega - \omega_0) - \delta(\omega + \omega_0)]$	$a_1 = -a_{-1} = \frac{1}{2j}$ $a_k = 0$ , otherwise
$x(t) = 1$	$2\pi \delta(\omega)$	$a_0 = 1$ , $a_k = 0$ , $k \neq 0$ (this is the Fourier series representation for any choice of $T > 0$ )
Periodic square wave $x(t) = \begin{cases} 1, &  t  < T_1 \\ 0, & T_1 <  t  \leq \frac{T}{2} \end{cases}$ and $x(t+T) = x(t)$	$\sum_{k=-\infty}^{+\infty} \frac{2 \sin k\omega_0 T_1}{k} \delta(\omega - k\omega_0)$	$\frac{\omega_0 T_1}{\pi} \operatorname{sinc}\left(\frac{k\omega_0 T_1}{\pi}\right) = \frac{\sin k\omega_0 T_1}{k\pi}$
$\sum_{n=-\infty}^{+\infty} \delta(t - nT)$	$\frac{2\pi}{T} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{T}\right)$	$a_k = \frac{1}{T}$ for all $k$
$x(t) \begin{cases} 1, &  t  < T_1 \\ 0, &  t  > T_1 \end{cases}$	$\frac{2 \sin \omega T_1}{\omega}$	—
$\frac{\sin Wt}{\pi t}$	$X(j\omega) = \begin{cases} 1, &  \omega  < W \\ 0, &  \omega  > W \end{cases}$	—
$\delta(t)$	1	—
$u(t)$	$\frac{1}{j\omega} + \pi \delta(\omega)$	—
$\delta(t - t_0)$	$e^{-j\omega t_0}$	—
$e^{-at} u(t), \Re\{a\} > 0$	$\frac{1}{a + j\omega}$	—
$te^{-at} u(t), \Re\{a\} > 0$	$\frac{1}{(a + j\omega)^2}$	—
$\frac{t^{n-1}}{(n-1)!} e^{-at} u(t), \Re\{a\} > 0$	$\frac{1}{(a + j\omega)^n}$	—

Table 5: **Properties of the Discrete-Time Fourier Transform**

$$x[n] = \frac{1}{2\pi} \int_{2\pi} X(e^{j\omega}) e^{j\omega n} d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{+\infty} x[n] e^{-j\omega n}$$

Property	Aperiodic Signal	Fourier transform
Linearity	$x[n]$ $y[n]$ $ax[n] + by[n]$	$X(e^{j\omega})$ $Y(e^{j\omega})$ $aX(e^{j\omega}) + bY(e^{j\omega})$
Time-Shifting	$x[n - n_0]$	$e^{-j\omega n_0} X(e^{j\omega})$
Frequency-Shifting	$e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$
Conjugation	$x^*[n]$	$X^*(e^{-j\omega})$
Time Reversal	$x[-n]$	$X(e^{-j\omega})$
Time Expansions	$x_{(k)}[n] = \begin{cases} x[n/k], & \text{if } n = \text{multiple of } k \\ 0, & \text{if } n \neq \text{multiple of } k \end{cases}$	$X(e^{jk\omega})$
Convolution	$x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi} \int_{2\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)})d\theta$
Differencing in Time	$x[n] - x[n - 1]$	$(1 - e^{-j\omega})X(e^{j\omega})$
Accumulation	$\sum_{k=-\infty}^n x[k]$	$\frac{1}{1 - e^{-j\omega}} X(e^{j\omega})$ $+ \pi X(e^{j0}) \sum_{k=-\infty}^{+\infty} \delta(\omega - 2\pi k)$
Differentiation in Frequency	$nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$
Conjugate Symmetry for Real Signals	$x[n]$ real	$\begin{cases} X(e^{j\omega}) = X^*(e^{-j\omega}) \\ \Re\{X(e^{j\omega})\} = \Re\{X(e^{-j\omega})\} \\ \Im\{X(e^{j\omega})\} = -\Im\{X(e^{-j\omega})\} \\  X(e^{j\omega})  =  X(e^{-j\omega})  \\ \angle X(e^{j\omega}) = -\angle X(e^{-j\omega}) \end{cases}$
Symmetry for Real, Even Signals	$x[n]$ real and even	$X(e^{j\omega})$ real and even
Symmetry for Real, Odd Signals	$x[n]$ real and odd	$X(e^{j\omega})$ purely imaginary and odd
Even-odd Decomposition of Real Signals	$x_e[n] = \mathcal{E}\{x[n]\}$ [ $x[n]$ real ] $x_o[n] = \mathcal{O}\{x[n]\}$ [ $x[n]$ real ]	$\Re\{X(e^{j\omega})\}$ $j\Im\{X(e^{j\omega})\}$

Parseval's Relation for Aperiodic Signals

$$\sum_{n=-\infty}^{+\infty} |x[n]|^2 = \frac{1}{2\pi} \int_{2\pi} |X(e^{j\omega})|^2 d\omega$$

Table 6: Basic Discrete-Time Fourier Transform Pairs

Signal	Fourier transform	Fourier series coefficients (if periodic)
$\sum_{k=\langle N \rangle} a_k e^{jk(2\pi/N)n}$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k$
$e^{j\omega_0 n}$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - \omega_0 - 2\pi l)$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} 1, & k = m, m \pm N, m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\cos \omega_0 n$	$\pi \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) + \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi m}{N}$ $a_k = \begin{cases} \frac{1}{2}, & k = \pm m, \pm m \pm N, \pm m \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$\sin \omega_0 n$	$\frac{\pi}{j} \sum_{l=-\infty}^{+\infty} \{\delta(\omega - \omega_0 - 2\pi l) - \delta(\omega + \omega_0 - 2\pi l)\}$	(a) $\omega_0 = \frac{2\pi r}{N}$ $a_k = \begin{cases} \frac{1}{2j}, & k = r, r \pm N, r \pm 2N, \dots \\ -\frac{1}{2j}, & k = -r, -r \pm N, -r \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$ (b) $\frac{\omega_0}{2\pi}$ irrational $\Rightarrow$ The signal is aperiodic
$x[n] = 1$	$2\pi \sum_{l=-\infty}^{+\infty} \delta(\omega - 2\pi l)$	$a_k = \begin{cases} 1, & k = 0, \pm N, \pm 2N, \dots \\ 0, & \text{otherwise} \end{cases}$
Periodic square wave $x[n] = \begin{cases} 1, &  n  \leq N_1 \\ 0, & N_1 <  n  \leq N/2 \end{cases}$ and $x[n + N] = x[n]$	$2\pi \sum_{k=-\infty}^{+\infty} a_k \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{\sin[(2\pi k/N)(N_1 + \frac{1}{2})]}{N \sin[2\pi k/2N]}$ , $k \neq 0, \pm N, \pm 2N, \dots$ $a_k = \frac{2N_1 + 1}{N}$ , $k = 0, \pm N, \pm 2N, \dots$
$\sum_{k=-\infty}^{+\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{+\infty} \delta\left(\omega - \frac{2\pi k}{N}\right)$	$a_k = \frac{1}{N}$ for all $k$
$a^n u[n]$ , $ a  < 1$	$\frac{1}{1 - ae^{-j\omega}}$	—
$x[n] \begin{cases} 1, &  n  \leq N_1 \\ 0, &  n  > N_1 \end{cases}$	$\frac{\sin[\omega(N_1 + \frac{1}{2})]}{\sin(\omega/2)}$	—
$\frac{\sin Wn}{\pi n} = \frac{W}{\pi} \text{sinc}\left(\frac{Wn}{\pi}\right)$ $0 < W < \pi$	$X(\omega) = \begin{cases} 1, & 0 \leq  \omega  \leq W \\ 0, & W <  \omega  \leq \pi \end{cases}$ $X(\omega)$ periodic with period $2\pi$	—
$\delta[n]$	1	—
$u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{+\infty} \pi \delta(\omega - 2\pi k)$	—
$\delta[n - n_0]$	$e^{-j\omega n_0}$	—
$(n + 1)a^n u[n]$ , $ a  < 1$	$\frac{1}{(1 - ae^{-j\omega})^2}$	—
$\frac{(n + r - 1)!}{n!(r - 1)!} a^n u[n]$ , $ a  < 1$	$\frac{1}{(1 - ae^{-j\omega})^r}$	—

Table 7: Properties of the Laplace Transform

Property	Signal	Transform	ROC
	$x(t)$	$X(s)$	$R$
	$x_1(t)$	$X_1(s)$	$R_1$
	$x_2(t)$	$X_2(s)$	$R_2$
Linearity	$ax_1(t) + bx_2(t)$	$aX_1(s) + bX_2(s)$	At least $R_1 \cap R_2$
Time shifting	$x(t - t_0)$	$e^{-st_0}X(s)$	$R$
Shifting in the $s$ -Domain	$e^{s_0t}x(t)$	$X(s - s_0)$	Shifted version of $R$ [i.e., $s$ is in the ROC if $(s - s_0)$ is in $R$ ]
Time scaling	$x(at)$	$\frac{1}{ a }X\left(\frac{s}{a}\right)$	“Scaled” ROC (i.e., $s$ is in the ROC if $(s/a)$ is in $R$ )
Conjugation	$x^*(t)$	$X^*(s^*)$	$R$
Convolution	$x_1(t) * x_2(t)$	$X_1(s)X_2(s)$	At least $R_1 \cap R_2$
Differentiation in the Time Domain	$\frac{d}{dt}x(t)$	$sX(s)$	At least $R$
Differentiation in the $s$ -Domain	$-tx(t)$	$\frac{d}{ds}X(s)$	$R$
Integration in the Time Domain	$\int_{-\infty}^t x(\tau)d(\tau)$	$\frac{1}{s}X(s)$	At least $R \cap \{\Re\{s\} > 0\}$

Initial- and Final Value Theorems

If  $x(t) = 0$  for  $t < 0$  and  $x(t)$  contains no impulses or higher-order singularities at  $t = 0$ , then

$$x(0^+) = \lim_{s \rightarrow \infty} sX(s)$$

If  $x(t) = 0$  for  $t < 0$  and  $x(t)$  has a finite limit as  $t \rightarrow \infty$ , then

$$\lim_{t \rightarrow \infty} x(t) = \lim_{s \rightarrow 0} sX(s)$$

Table 8: Laplace Transforms of Elementary Functions

Signal	Transform	ROC
1. $\delta(t)$	1	All $s$
2. $u(t)$	$\frac{1}{s}$	$\Re\{s\} > 0$
3. $-u(-t)$	$\frac{1}{s}$	$\Re\{s\} < 0$
4. $\frac{t^{n-1}}{(n-1)!}u(t)$	$\frac{1}{s^n}$	$\Re\{s\} > 0$
5. $-\frac{t^{n-1}}{(n-1)!}u(-t)$	$\frac{1}{s^n}$	$\Re\{s\} < 0$
6. $e^{-\alpha t}u(t)$	$\frac{1}{s + \alpha}$	$\Re\{s\} > -\alpha$
7. $-e^{-\alpha t}u(-t)$	$\frac{1}{s + \alpha}$	$\Re\{s\} < -\alpha$
8. $\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(t)$	$\frac{1}{(s + \alpha)^n}$	$\Re\{s\} > -\alpha$
9. $-\frac{t^{n-1}}{(n-1)!}e^{-\alpha t}u(-t)$	$\frac{1}{(s + \alpha)^n}$	$\Re\{s\} < -\alpha$
10. $\delta(t - T)$	$e^{-sT}$	All $s$
11. $[\cos \omega_0 t]u(t)$	$\frac{s}{s^2 + \omega_0^2}$	$\Re\{s\} > 0$
12. $[\sin \omega_0 t]u(t)$	$\frac{\omega_0}{s^2 + \omega_0^2}$	$\Re\{s\} > 0$
13. $[e^{-\alpha t} \cos \omega_0 t]u(t)$	$\frac{s + \alpha}{(s + \alpha)^2 + \omega_0^2}$	$\Re\{s\} > -\alpha$
14. $[e^{-\alpha t} \sin \omega_0 t]u(t)$	$\frac{\omega_0}{(s + \alpha)^2 + \omega_0^2}$	$\Re\{s\} > -\alpha$
15. $u_n(t) = \frac{d^n \delta(t)}{dt^n}$	$s^n$	All $s$
16. $u_{-n}(t) = \underbrace{u(t) * \dots * u(t)}_{n \text{ times}}$	$\frac{1}{s^n}$	$\Re\{s\} > 0$



## z-TRANSFORM PROPERTIES

The index-domain signal is  $x[n]$  for  $-\infty < n < \infty$ ; and the  $z$ -transform is:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \iff x[n] = \frac{1}{2\pi j} \oint X(z) z^n \frac{dz}{z}$$

The ROC is the set of complex numbers  $z$  where the  $z$ -transform sum converges.

Signal: $x[n] \quad -\infty < n < \infty$	$z$ -Transform: $X(z)$	Region of Convergence
$x[n], x_1[n]$ and $x_2[n]$	$X(z), X_1(z)$ and $X_2(z)$	$R_x, R_1$ and $R_2$
$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	contains $R_1 \cap R_2$
$x[n - n_0]$	$z^{-n_0} X(z)$	$R_x$ except for the possible addition or deletion of $z = 0$ or $z = \infty$
$z_0^n x[n]$	$X(z/z_0)$	$ z_0  R_x$
$nx[n]$	$-z \frac{dX(z)}{dz}$	$R_x$ except for the possible addition or deletion of $z = 0$ or $z = \infty$
$x^*[n]$	$X^*(z^*)$	$R_x$
$\Re\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	contains $R_x$
$\Im\{x[n]\}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	contains $R_x$
$x[-n]$	$X(1/z)$	$1/R_x = \{z : z^{-1} \in R_x\}$
$x_1[n] * x_2[n]$	$X_1(z) \cdot X_2(z)$	contains $R_1 \cap R_2$
$x_1[n] \cdot x_2[n]$	$\frac{1}{2\pi j} \oint X_1(v) X_2(z/v) \frac{dv}{v}$	contains $R_1 R_2$
Parseval's Theorem:	$\sum_{n=-\infty}^{\infty} x_1[n] x_2^*[n] = \frac{1}{2\pi j} \oint X_1(v) X_2^*(1/v^*) \frac{dv}{v}$	
Initial Value Theorem:	$x[n] = 0, \text{ for } n < 0 \implies \lim_{z \rightarrow \infty} X(z) = x[0]$	

## z-TRANSFORM PAIRS

The index-domain signal is  $x[n]$  for  $-\infty < n < \infty$ ; and the  $z$ -transform is:

$$X(z) = \sum_{n=-\infty}^{\infty} x[n] z^{-n} \quad \Longleftrightarrow \quad x[n] = \frac{1}{2\pi j} \oint X(z) z^n \frac{dz}{z}$$

The ROC is the set of complex numbers  $z$  where the  $z$ -transform sum converges.

Signal: $x[n]$ $-\infty < n < \infty$	$z$ -Transform: $X(z)$	Region of Convergence
$\delta[n]$	1	All $z$
$\delta[n - n_0]$	$z^{-n_0}$	$ z  > 0$ , if $n_0 > 0$ $ z  < \infty$ , if $n_0 < 0$
$u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
$-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z  < 1$
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z  >  a $
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z  <  a $
$n a^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  >  a $
$-n a^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  <  a $
$(n + 1) a^n u[n]$	$\frac{1}{(1 - az^{-1})^2}$	$ z  >  a $
$[\cos \omega_0 n] u[n]$	$\frac{1 - [\cos \omega_0] z^{-1}}{1 - 2[\cos \omega_0] z^{-1} + z^{-2}}$	$ z  > 1$
$[\sin \omega_0 n] u[n]$	$\frac{[\sin \omega_0] z^{-1}}{1 - 2[\cos \omega_0] z^{-1} + z^{-2}}$	$ z  > 1$
$[r^n \cos \omega_0 n] u[n]$	$\frac{1 - [r \cos \omega_0] z^{-1}}{1 - 2r[\cos \omega_0] z^{-1} + r^2 z^{-2}}$	$ z  >  r $
$[r^n \sin \omega_0 n] u[n]$	$\frac{[r \sin \omega_0] z^{-1}}{1 - 2r[\cos \omega_0] z^{-1} + r^2 z^{-2}}$	$ z  >  r $
$x[n] = \begin{cases} a^n, & 0 \leq n < L \\ 0, & \text{otherwise} \end{cases}$	$\frac{1 - a^L z^{-L}}{1 - az^{-1}}$	$ z  > 0$

## PROPERTIES of the DFT

The index-domain signal is  $x[n]$  for  $n = 0, 1, 2, \dots, N-1$ ; and the frequency domain values are  $X[k]$  for  $k = 0, 1, 2, \dots, N-1$ . Outside the range  $[0, N-1]$ , the values of  $x[n]$  and  $X[k]$  are periodic.

$$\text{Analysis Equation: } X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk} \quad (1)$$

$$\text{Synthesis Equation: } x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-nk} \quad (2)$$

$$\text{Exponential: } W_N = e^{-j2\pi/N} \quad (3)$$

$N$ -point signal: $x[n], n = 0, 1, \dots, N-1$	$N$ -point DFT: $X[k], k = 0, 1, \dots, N-1$
$ax_1[n] + bx_2[n]$	$aX_1[k] + bX_2[k]$ (Linearity)
$x[n] = W_N^{-\ell_o n} = e^{+j2\pi\ell_o n/N}$	$X[k] = N \delta[(k - \ell_o) \bmod N]$
$y[n] = X[k]  _{k \leftarrow n}$	$Y[k] = N \cdot x[(-k) \bmod N]$ (Duality)
$y[n] = \frac{1}{N} \cdot X[(-n) \bmod N]$	$Y[k] = x[n]  _{n \leftarrow k}$ (Duality)
$x[(n - n_o) \bmod N]$	$W_N^{n_o k} X[k]$
$W_N^{-\ell_o n} x[n]$	$X[(k - \ell_o) \bmod N]$
$x[n] \textcircled{N} h[n] = \sum_{\ell=0}^{N-1} x[\ell] h[(n - \ell) \bmod N]$	$X[k] \cdot H[k]$
$x[n] w[n]$ (windowing)	$\frac{1}{N} \sum_{\ell=0}^{N-1} X[\ell] W[(k - \ell) \bmod N]$
$x^*[n]$	$X^*[(-k) \bmod N]$
$x^*[(-n) \bmod N]$	$X^*[k]$
$\Re\{x[n]\}$	$X_{cs}[k] = \frac{1}{2} \{X[k \bmod N] + X^*[(-k) \bmod N]\}$
$j\Im\{x[n]\}$	$X_{cas}[k] = \frac{1}{2} \{X[k \bmod N] - X^*[(-k) \bmod N]\}$
$x_{cs}[n] = \frac{1}{2} \{x[n \bmod N] + x^*[(-n) \bmod N]\}$	$\Re\{X[k]\}$ (cs = conjugate-sym)
$x_{cas}[n] = \frac{1}{2} \{x[n \bmod N] - x^*[(-n) \bmod N]\}$	$j\Im\{X[k]\}$ (cas = conj-anti-sym)
Parseval's Theorem: $N \sum_{n=0}^{N-1}  x[n] ^2 = \sum_{k=0}^{N-1}  X[k] ^2$	
The following properties apply when $x[n]$ is purely real:	
Conjugate Symmetry	$X[k] = X^*[(-k) \bmod N]$
Real part of $X[k]$ is even	$\Re\{X[k]\} = \Re\{X[(-k) \bmod N]\}$
Imaginary part of $X[k]$ is odd	$\Im\{X[k]\} = -\Im\{X[(-k) \bmod N]\}$
$ X[k] $ (Magnitude) is even	
$\arg\{X[k]\}$ (Phase) is odd	