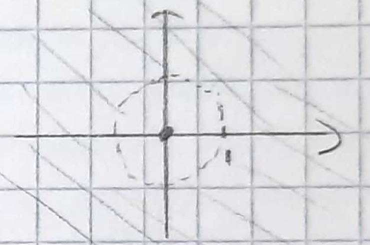
67) Desarrollo alrededor de ∞

67.1) $F(z) = \dfrac{1}{z(z-1)}$, singularidades: 0, 1



$$z_0 = 1 \rightarrow f(z) = \frac{1}{(z-0)(z-1)} = \frac{A}{z} + \frac{B}{z-1} = \frac{A(z-1)+B(z)}{z(z-1)} = \frac{z(A+B) - A}{z(z-1)}$$

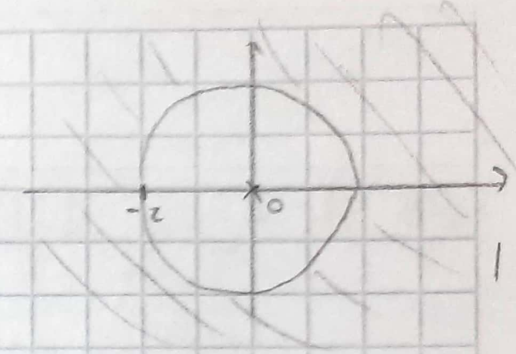
$$\begin{cases} (A+B) = 0 \rightarrow A = -B \\ -A = 1 \rightarrow A = -1 \;,\; B = 1 \end{cases} \rightarrow f(z) = -\overbrace{\frac{1}{z}}^{F_1(z)} + \overbrace{\frac{1}{z-1}}^{F_2(z)}$$

$$F_1(z) = \frac{1}{z-1+1} = 1 + \frac{1}{z-1} = 1 + \frac{1}{z}\,\frac{1}{1-\frac{1}{z}} = 1 + \frac{1}{z}\sum_{n=0}^{\infty}\frac{1^n}{z^n} \;,\quad \left|\frac{1}{z}\right| < 1 \rightarrow 1 < |z| \checkmark$$

$$= 1 + \sum_{n=0}^{\infty}\frac{1^n}{z^{n+1}}$$

$$\Rightarrow f(z) = \sum_{n=0}^{\infty}\frac{1^n}{z^{n+1}} - 1 - \sum_{n=0}^{\infty}\frac{1^n}{z^{n+1}} = \boxed{-1} \quad ?$$

67.5] $F(z) = \frac{(z-1)}{z(z+2)}$ Singularidades : 0, −2



$|z| > 2$

$$= \frac{(z-1)}{(z-0)(z+2)} = \frac{A}{z} + \frac{B}{z+2} = \frac{A(z+2)+Bz}{z(z+2)} = \frac{z(A+B)+2A}{z(z+2)}$$

$$\begin{cases} A+B = 1 \rightarrow B = \frac{3}{2} \\ 2A = -1 \rightarrow A = -\frac{1}{2} \end{cases}$$

$$F(z) = -\frac{1}{2}\cdot\overbrace{\frac{1}{z}}^{F_1(z)} + \frac{3}{2}\cdot\overbrace{\frac{1}{z+2}}^{F_2(z)}$$

$$F_1(z) = \frac{1}{z+2-2} = -\frac{1}{2} + \frac{1}{z+2} = -\frac{1}{z} + \frac{1}{z}\,\frac{1}{1-\left(-\frac{2}{z}\right)} = -\frac{1}{2} + \frac{1}{z}\sum_{n=0}^{\infty}\left(-\frac{2}{z}\right)^n, \quad \left|\frac{2}{z}\right| < 1 \rightarrow 2 < |z|$$

$$= -\frac{1}{2} + \sum_{n=0}^{\infty}\frac{(-2)^n}{z^{n+1}}$$

$$F_2(z) = \frac{1}{z+2} = \sum_{n=0}^{\infty}\frac{(-2)^n}{z^{n+1}} \quad |z|>2 \Rightarrow \boxed{F(z) = -\frac{1}{2}\left[-\frac{1}{2} + \sum_{n=0}^{\infty}\frac{(-2)^n}{z^{n+1}}\right] + \frac{3}{2}\left[\sum_{n=0}^{\infty}\frac{(-2)^n}{z^{n+1}}\right] \quad \text{CV } 2<|z|}$$