FINAL 9 A) Resolver $\underbrace{\int_0^{+\infty} f(t)\cdot \operatorname{sen}(\omega t)\,dt}_{F_s(\omega)} = \underbrace{\frac{1-\cos(\omega)}{\omega}}_{F_s(\omega)}$ (14.1)

Jude

(calcular $f(t)$)

- $\boxed{TF}$

$$F(\omega) = \int_{-\infty}^{+\infty} f(t)\, e^{-i\omega t}\,dt = \underbrace{\int_{-\infty}^{+\infty} f(t)\cdot \cos(\omega t)\,dt}_{2F_c(\omega)} - i\underbrace{\int_{-\infty}^{+\infty} f(t)\operatorname{sen}(\omega t)\,dt}_{2F_s(\omega)}$$

- la integral se parece mucho a la TF seno de una función impar $\Rightarrow$ digo que la $f(t)$ buscada es impar $\Rightarrow$

$$F(\omega) = -2i\,F_s(\omega) = -2i\int_0^{+\infty} f(t)\operatorname{sen}(\omega t) = 2\left(\frac{1-\cos(\omega)}{\omega}\right)$$

- $\boxed{ATF}$ (para obtener $f(t)$)

$$f^*(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} F(\omega)\cdot e^{i\omega t}\,d\omega = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{2}{\omega}\cdot e^{i\omega t}\,d\omega - \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{2\cos(\omega)}{\omega}\,e^{i\omega t}\,d\omega$$

TIP de Ameto Ib: como tengo una trigonométrica multiplicada por una exponencial, no reemplazo $\cos(\omega t) = e^{iz}$, sino que uso la forma exponencial del $\cos(\omega) = \frac{e^{i\omega}+e^{-i\omega}}{2}$

$$f^*(t) = \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{2}{\omega}\,e^{i\omega t}\,d\omega - \frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{\cancel{2}(e^{i\omega}+e^{-i\omega})}{\cancel{2}\omega}\cdot e^{i\omega t}\,d\omega =$$

$$= \underbrace{\frac{1}{\pi}\int_{-\infty}^{+\infty}\frac{e^{i\omega t}}{\omega}\,d\omega}_{I} - \underbrace{\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{e^{i\omega(1+t)}}{\omega}\,d\omega}_{II} - \underbrace{\frac{1}{2\pi}\int_{-\infty}^{+\infty}\frac{e^{i\omega(t-1)}}{\omega}\,d\omega}_{III}$$

- casos a analizar cuando paso al plano complejo:
- $t=0 \longrightarrow t=0$
- $1+t=0 \longrightarrow t=-1$
- $t-1=0 \longrightarrow t=1$

$\Rightarrow$ 4 casos
- $t<-1$
- $-1<t<0$
- $0<t<1$
- $t>1$

- al pasar al plano complejo, las singularidades son $z=0$

$\lim_{z\to 0} z\,\frac{e^{izt}}{z} = \boxed{1}$; $\lim_{z\to 0} z\,\frac{e^{iz(1+t)}}{z} = \boxed{1}$; $\lim_{z\to 0} z\,\frac{e^{iz(t-1)}}{z} = \boxed{1}$

para los 3 fuecos es P1° y $\underline{R_s = 1}$

Teoremas

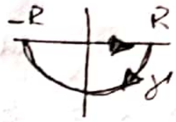
- **Teorema de Jordan** (para acotar integral multiplicado por exponencial)

  - caso 1 $(m>0)$

$$\left.\begin{array}{l} H_1)\ \lim\limits_{z\to\infty} f(z)=0 \\ H_2)\ m>0 \\ H_3)\ \Gamma_R: z=Re^{i\varphi},\ \varphi\in(0,\pi) \end{array}\right\} \Rightarrow \lim_{R\to\infty}\int_{\Gamma_R} f(z)\,e^{imz}\,dz=0$$

  - caso 2 $(m<0)$

$$\left.\begin{array}{l} H_1)\ \lim\limits_{z\to\infty} f(z)=0 \\ H_2)\ m<0 \\ H_3)\ \Gamma_R: z=R\cdot e^{i\varphi},\ \varphi\in(\pi,2\pi) \end{array}\right\} \Rightarrow \lim_{R\to\infty}\int_{\Gamma_R} f(z)\,e^{+imz}\,dz=0$$



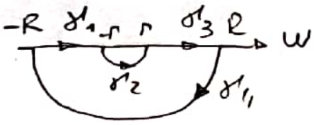
- **Teorema de la integral en un arco que rodea a un P1°**

$$\left.\begin{array}{l} H_1)\ z_0\in P1^\circ \text{ de } f(z) \\ H_2)\ \gamma: z=z_0+r\cdot e^{i\varphi};\ \varphi\in(a;a+\alpha) \end{array}\right\} \Rightarrow \lim_{r\to 0}\int_{\gamma} f(z)\cdot dz = i\,\alpha\cdot \operatorname{Res}[f(z);z_0]$$

Analizo los 4 casos

$\boxed{t<-1}$ → I → Jordan caso 2
→ II → Jordan caso 2
→ III → Jordan caso 2

$\boxed{\text{I}}$



$$\oint_{\Gamma} = \int_{\gamma_1} + \int_{\gamma_2} + \int_{\gamma_3} + \int_{\gamma_4} = 2\pi i \operatorname{Res} = 0 \ \left(\begin{array}{c}\text{no encierra}\\ \text{sing}\end{array}\right)$$

$$\left.\begin{array}{l} \gamma_1: \displaystyle\int_{-R}^{-r}\frac{e^{iwt}}{w}\,dw \xrightarrow[r\to 0]{R\to\infty} \int_{-\infty}^{0}\frac{e^{iwt}}{w}\,dw \\ \gamma_3: \displaystyle\int_{r}^{R}\frac{e^{iwt}}{w}\,dw \longrightarrow \int_{0}^{+\infty}\frac{e^{iwt}}{w}\,dw \end{array}\right\} VP\int_{-\infty}^{+\infty}\frac{e^{iwt}}{w}\,dw$$

γ_2: Teorema de la integral en un arco que rodea P1°.

$$\left.\begin{array}{l} H_1)\ z=0\in P1^\circ \text{ de } f(z)=\dfrac{e^{izt}}{z} \\ H_2)\ \gamma: z=0+r\cdot e^{i\varphi}\ \ \varphi\in(\pi;2\pi) \end{array}\right\} \lim_{r\to 0}\int_{\gamma_3} f(z)\,dz = i\,(2\pi-\pi)\cdot 1 = \boxed{i\pi}$$

γ_4: Jordan

$$\left.\begin{array}{l} H_1)\ \lim\limits_{z\to\infty}\dfrac{1}{z}=0 \\ H_2)\ m<0\ \ (t<0) \\ H_3)\ \Gamma_R: z=R\cdot e^{i\varphi};\ \varphi(\pi,2\pi) \end{array}\right\} \Rightarrow \lim_{R\to\infty}\int_{\gamma_4} f(z)\cdot e^{imz}\,dz = \boxed{0}$$

$$\Rightarrow \int_{\gamma_1} + \int_{\gamma_2} + \int_{\gamma_3} + \int_{\gamma_4} = 0 \Rightarrow VP\int_{-\infty}^{+\infty}\frac{e^{iwt}}{w}\,dw = \boxed{-i\pi}$$

$\boxed{II}$ idem $\Rightarrow VP\int_{-\infty}^{+\infty} \frac{e^{i\omega(1+t)}}{\omega}\,d\omega = \boxed{-i\pi}$

$\boxed{III}$ idem $\rightarrow VP\int_{-\infty}^{+\infty} \frac{e^{i\omega(t-1)}}{\omega}\,d\omega = \boxed{-i\pi}$

$\Rightarrow \boxed{f^*(t)} = \frac{1}{\pi}\boxed{I} - \frac{1}{2\pi}\boxed{II} - \frac{1}{2\pi}\boxed{III}$

$= \frac{1}{\pi}(-i\pi) - \frac{1}{2\pi}(-i\pi) - \frac{1}{2\pi}(-i\pi) = \cancel{\boxed{0}}$

$= -i + \frac{1}{2}i + \frac{1}{2}i = \boxed{0}$

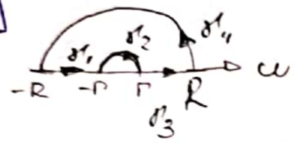
$\boxed{-1 < t < 0}$ → I: Jordan caso 2

↳ II Jordan caso 1

↳ III Jordan caso 2

$\boxed{I}$ idem que $\boxed{I}$ para caso $t < -1 \Rightarrow I = -i\pi$

$\boxed{II}$



$\oint_{\Gamma} = \int_{\gamma_1} + \int_{\gamma_2} + \int_{\gamma_3} + \int_{\gamma_4} = 2\pi i \cdot \text{Res} = 0$ (no encierra sing)

$\cdot\ \gamma_1: \int_{-R}^{-r} \frac{e^{i\omega(1+t)}}{\omega}\,d\omega \xrightarrow[r\to 0]{R\to\infty} \int_{-\infty}^{0} \frac{e^{i\omega(1+t)}}{\omega}\,d\omega$

$\cdot\ \gamma_3: \int_{r}^{R} \frac{e^{i\omega(1+t)}}{\omega}\,d\omega \xrightarrow[r\to 0]{R\to\infty} \int_{0}^{+\infty} \frac{e^{i\omega(1+t)}}{\omega}\,d\omega$

$\Bigg\} \ VP\int_{-\infty}^{+\infty} \frac{e^{i\omega(1+t)}}{\omega}\,d\omega$

$\cdot\ \gamma_2$: Teorema de la integral en un arco que rodea Pic.

$H_1)\ z = 0 \in P1^{\circ}$ de $f(z) = \frac{e^{i\omega(1+t)}}{\omega}$

$H_2)\ \gamma: z = 0 + r\,e^{i\varphi} \quad \varphi \in (\pi; 0)$ $\Bigg\} \Rightarrow \lim_{r\to 0} \int_{\gamma_3} f(z)\,dz = i(0-\pi)\cdot 1 = \boxed{-i\pi}$

$\cdot\ \gamma_4$: Jordan $(m > 0)$

$H_1)\ \lim_{z\to\infty} \frac{1}{z} = 0$

$H_2)\ m > 0$

$H_3)\ \gamma_4: z = R\cdot e^{i\varphi};\ \varphi \in (0,\pi)$ $\Bigg\} \Rightarrow \lim_{R\to\infty} \int_{\gamma_4} f(z)\cdot e^{imt}\,dz = \boxed{0}$

$\Rightarrow VP\int_{-\infty}^{+\infty} \frac{e^{i\omega(1+t)}}{\omega}\,d\omega = \boxed{i\pi}$

$\boxed{III}$ idem que $\boxed{III}$ para caso $t < -1 \Rightarrow III = \boxed{-i\pi}$

$\Rightarrow \boxed{f^*(t)} = \frac{1}{\pi}\boxed{I} - \frac{1}{2\pi}\boxed{II} - \frac{1}{2\pi}\boxed{III}$

$= \frac{1}{\pi}(-i\pi) - \cancel{\frac{1}{2\pi}(i\pi)} - \cancel{\frac{1}{2\pi}(-i\pi)} = \boxed{-i}$

$\boxed{0 < t < 1}$ → I: Jordan caso 1
→ II: Jordan caso 1
→ III: Jordan caso 2

$\boxed{I} = i\pi$

$\boxed{II} = i\pi$

$\boxed{III} = -i\pi$

$$\boxed{f^{-1}(t)} = \frac{1}{\pi}(i\pi) - \frac{1}{2\pi}(i\pi) - \frac{1}{2\pi}(-i\pi) = \boxed{i}$$

$\boxed{t > 1}$ → I: Jordan caso 1
→ II Jordan caso 1
→ III Jordan caso 1

$\boxed{I} = i\pi$

$\boxed{II} = i\pi$

$\boxed{III} = i\pi$

$$\boxed{f^{-1}(t)} = \frac{1}{\pi}(i\pi) - \frac{1}{2\pi}(i\pi) - \frac{1}{2\pi}(i\pi) = \boxed{0}$$